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SHOCK WAVES IN UNSTEADY FLOWS

Gerard Tavera, Henri Burnage

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16. Abstract

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SHOCK WAVES IN UNSTEADY FLOWS /571*

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ABSTRACT

The space-time motion of a shock wave in an unsteady flow is taken into account through its kinematics. The space is divided into an upstream and downstream region and the shock is replaced by equivalent distribution of sources of mass, momentum and energy expressed as functions of the upstream quantities and the shock motion.

INTRODUCTION

The purpose of this study is to replace an unsteady shock wave by an adequate distribution of mass sources, momentum sources, and energy sources. In this way, in the downstream area, we can reconstitute a flow identical to the one in which exists in reality.

Kinematics of the shock wave. in a fixed reference system, we will consider a shock wave considered thin and distributed by means of the following relationship

$$f(x_i, t) = 0,$$

where x_i represents a point of the shock wave and t is the time.

We assume that the space is then divided by the shock wave into two regions, an upstream region (subscript 1) in

* Numbers in margin indicate pagination of foreign text.

which we have $f < 0$ and a downstream region (subscript 2) where $f > 0$. Also, we assume that $f(x_i t)$ can be differentiated.

Let us now consider a motion of the components dx_i of a point C of the shock wave, and then we have

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x_i}\frac{dx_i}{dt} = 0.$$

which can be written as:

$$\frac{\partial f}{\partial t} + H.u_n = 0 \Rightarrow u_n = -\frac{1}{H}\frac{\partial f}{\partial t}, \tag{1}$$

with

$$H^2 = \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_i}, \; n_i = \frac{1}{H}\frac{\partial f}{\partial x_i}$$

normal to the shock point C. $dx_i/dt = u_i$ is the component of the velocity at the point C. Use of n is the component perpendicular to the velocity at the point C.

Conservation equations. Let $\varphi(x_i, t)$ be a field assumed to be discontinuous through a shock wave represented at a point P by a relationship having the form

$$\varphi(x_i, t) = h(-f)\varphi_1 + h(f)\varphi_2,$$

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where h(f) is the Heaviside scale. φ_1 and φ_2 are values of the field when the point P with coordinates x_i is located in the upstream or downstream region respectively. At any point of the flow and outside of these singularities, the continuity equation is written with the usual notation as follows

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}\rho v_j = 0.$$

By replacing φ with ρ and ρv_j, we find that

$$\rho = h(-f)\rho_1 + h(f)\rho_2; \qquad \rho v_j = h(-f)(\rho v_j)_1 + h(f)(\rho v_j)_2.$$

By substituting in the continuity equation we find

$$\left[\rho_1 \frac{\partial f}{\partial t} + (\rho v_j)_1 \frac{\partial f}{\partial x_j}\right]\delta(f) = \left[\rho_2 \frac{\partial f}{\partial t} + (\rho v_j)_2 \frac{\partial f}{\partial x_j}\right]\delta(f), \tag{2}$$

where $\delta(f)$ is the Dirac measure carried by the shock.

By using (1) we find the relationship which is well known ([1]):

$$\rho_1(v_{n,1} - u_n) = \rho_2(v_{n,2} - u_n),$$

which expresses the conservation of the flow rate through a shock in motion. Also, by multiplying the continuity equation with h(f) we obtain

$$\frac{\partial}{\partial t} h(f)\rho + \frac{\partial}{\partial x_j} h(f)\rho v_j = \rho_2(v_{n,2} - u_n)\delta(g),$$

where

$$dg/df = H \qquad \text{and} \qquad g = 0, \quad \text{si } f = 0$$

and by using the relationship (3), finally we find

$$\frac{\partial}{\partial t} h(f)\rho + \frac{\partial}{\partial x_j} h(f)\rho v_j = \rho_1(v_{n,1} - u_n)\delta(g),$$

this equation expresses the principal of conservation of mass in a downstream medium by considering the shock wave as a distribution of mass sources represented by the term on the right side.

By using the same procedure, the conservation equations of the momentum and energy can be respectively written using the following notation:

$$\frac{\partial}{\partial t}(h\rho v_i)+\frac{\partial}{\partial x_j}h(\rho v_i v_j-\sigma_{ij})=\{(\rho v_i)_1(v_{n,1}-u_n)-(\sigma_{ij})_1 \cdot n_j\}\delta(g).$$

where

$$\sigma_{ij}=-P\delta_{ij}+\tau_{ij}$$

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$$\frac{\partial}{\partial t}h\left[\rho\left(e+\frac{1}{2}v^2\right)\right]+\frac{\partial}{\partial x_j}h\left[\rho\left(e+\frac{v^2}{2}\right)v_j-\sigma_{ij}v_j+q_j\right]$$
$$=\left\{\rho_1\left(e_1+\frac{1}{2}v_1^2\right)(v_{n,1}-u_n)-n_i(\sigma_{ij}v_j)_1+(q_e)_1 n_e\right\}\delta(g).$$

The second terms of these equations represent the sources which are equivalent and which are arranged at every point of the shock, commonly expressed as a function of the upstream conditions and the kinematics of the shock wave. This represents an application of the Huyghens principle to the problem of interaction between the shock wave and an unsteady flow.

REFERENCES

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